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Title: A d-estimator for single-case designs

Authors and Affiliations:

William Shadish
University of California, Merced
School of Social Sciences, Humanities and Arts
5200 N. Lake Rd
Merced CA 95343
209-228-4372
wshadish@ucmerced.edu

Larry Hedges
Institute for Policy Research
Northwestern University
2040 N. Sheridan Road
Evanston, IL 60208
Telephone: (847) 491-8899
FAX: (847) 467-2459
Email: l-hedges@northwestern.edu

James Pustejovsky
Institute for Policy Research
Northwestern University
2040 N. Sheridan Road
Evanston, IL 60208
Telephone: (847) 491-8899
FAX: (847) 467-2459
Email: JamesPustejovsky2013@u.northwestern.edu

David Rindskopf
Educational Psychology
CUNY Graduate Center
365 Fifth Avenue
New York, NY 10016
drindskopf@gc.cuny.edu

Abstract Body
Limit 4 pages single-spaced.

Background / Context:
Description of prior research and its intellectual context.

Over the last 10 years, numerous authors have proposed effect size estimators for single-case designs. None, however, has been shown to be equivalent to the usual between-groups standardized mean difference statistic, sometimes called d. The present paper remedies that omission.

Purpose / Objective / Research Question / Focus of Study:
Description of the focus of the research.

Most effect size estimators for single-case designs use the within-person variance as the standardization factor for the effect size. But in the usual between-groups study, like a randomized experiment, the between-person variance is the standardization factor. Taking advantage of the fact that most studies that use single-case design methodology report results from more than one case, we show how a d-statistic can be developed that is equivalent to the between-groups estimator. The new d-estimator is complex, requiring knowledge of the autocorrelation, the within-person variability, and the between-person variability. We have written SPSS syntax for its use. .

Setting:
Description of the research location.
(May not be applicable for Methods submissions)

(Not applicable)

Population / Participants / Subjects:
Description of the participants in the study: who, how many, key features, or characteristics.
(May not be applicable for Methods submissions)

(Not applicable)

Intervention / Program / Practice:
Description of the intervention, program, or practice, including details of administration and duration.
(May not be applicable for Methods submissions)

(Not applicable)

Significance / Novelty of study:
Description of what is missing in previous work and the contribution the study makes.

No previous work has developed a d-statistic for single-case designs that is shown statistically to be equivalent to the usual between-groups d.

Statistical, Measurement, or Econometric Model:
Description of the proposed new methods or novel applications of existing methods.

It is difficult to summarize the statistic succinctly. Here is about as succinct as we can do. Define the total sample size as m. Then define the effect size estimate ES via

$$ES = \frac{\bar{D}}{S} \qquad (1)$$

where

$$\bar{D} = \frac{1}{m}\sum_{i=1}^{m}\left(\frac{1}{n}\sum_{t=n+1}^{2n} Y_{it} - \frac{1}{n}\sum_{t=1}^{n} Y_{it}\right),$$

$$S^2 = \frac{1}{2n(m-1)}\sum_{t=1}^{2n}\sum_{i=1}^{m}\left(Y_{it} - \bar{Y}_{\bullet t}\right)^2,$$

and $\bar{Y}_{\bullet t}$ is the mean across individuals at the t^{th} timepoint given by

$$\bar{Y}_{\bullet t} = \frac{1}{m}\sum_{i=1}^{m} Y_{it}\,.$$

It follows that $\bar{D}$ is an unbiased estimator of $(\mu^T - \mu^C)$ and S^2 is an unbiased estimate of the total variance $(\sigma^2 + \tau^2)$.

Under this model the variance of $\bar{D}$ is

$$\mathrm{V}\left\{\bar{D}\right\} = \frac{2(b_1 - c_1)\sigma^2}{m},$$

where b_p and c_p are functions of the autocorrelation ϕ and the phase length n defined in general as

$$b_p = \frac{1}{n} + \frac{2}{n^2}\sum_{t=1}^{n-1}\phi^{pt}\left(n-t\right)$$

and

$$c_p = \frac{1}{n^2}\sum_{t=1-n}^{n-1}\phi^{p(n+t)}\left(n-|t|\right).$$

Here we define where b_p and c_p in general because although the variance of $\bar{D}$ depends only on b_1 and c_1 (that is b_p and c_p for $p = 1$) we will need b_2 and c_2 later.

Under this model the variance of S^2 is

$$\mathrm{V}\left\{S^2\right\} = \frac{\left[\left(b_2 + c_2\right)\left(1-\rho\right)^2 + 2\left(b_1 + c_1\right)\rho\left(1-\rho\right) + 2\rho^2\right]\left(\sigma^2 + \tau^2\right)^2}{m-1},$$

where

$$\rho = \frac{\tau^2}{\tau^2 + \sigma^2} \tag{2}$$

is a kind of intraclass correlation that represents the between-person variance τ^2 as a fraction of the total variance (τ^2 +σ^2).

It follows from this (using the Box-Satterthwaite theorem) that the sampling distribution of ES is a constant k times a noncentral t-distribution with v degrees of freedom, where k is given by

$$k = \sqrt{\frac{\mathrm{V}\left\{\bar{D}\right\}}{\tau^2 + \sigma^2}} = \sqrt{\frac{2\left(b_1 - c_1\right)\left(1-\rho\right)}{m}}$$

and v is given by

$$v = \frac{2\left(m-1\right)}{\left(b_2 + c_2\right)\left(1-\rho\right)^2 + 2\left(b_1 + c_1\right)\rho\left(1-\rho\right) + 2\rho^2}. \tag{3}$$

If follows from results in Hedges (1981) that the bias in ES can be corrected by multiplying ES by the correction factor

$$\mathrm{J}\left(v\right) = 1 - \frac{3}{4v-1} \tag{4}$$

So that the effect size

$$G = \mathrm{J}\left(v\right)ES \tag{5}$$

is approximately an unbiased estimator of δ.

It also follows that the variance of G is approximately

$$\mathrm{V}\left\{G\right\} = \mathrm{J}\left(v\right)^2\left[\frac{vk^2}{v-2} + \delta^2\left(\frac{v}{v-2} - \frac{1}{\mathrm{J}\left(v\right)^2}\right)\right]. \tag{6}$$

A slightly simpler asymptotic approximation of the variance is

$$\mathrm{V}_{\mathrm{A}}\left\{G\right\} = \mathrm{J}\left(v\right)^2\left[k^2 + \frac{\delta^2}{2v}\right]. \tag{7}$$

Usefulness / Applicability of Method:
Demonstration of the usefulness of the proposed methods using hypothetical or real data.

We will illustrate the statistic with two examples, one from medicine and one from education. Both cases will compare results of our d-statistics applied to SCDs to d from comparable randomized designs.

Research Design:
Description of the research design (e.g., qualitative case study, quasi-experimental design, secondary analysis, analytic essay, randomized field trial).

(May not be applicable for Methods submissions)

(not applicable)

Data Collection and Analysis:
Description of the methods for collecting and analyzing data.
(May not be applicable for Methods submissions)

(not applicable)

Findings / Results:
Description of the main findings with specific details.
(May not be applicable for Methods submissions)

The results are already available for cases with some simplifying assumptions, and we hope full results will be available by the conference without those simplifying assumptions. Results suggest for the examples given that our d-statistic applied to single-case designs yields approximately the same effect size as the d-statistic from comparable randomized experiments.

Conclusions:
Description of conclusions, recommendations, and limitations based on findings.

The new d-statistic should be of general interest to analysts of SCDs who wish to compute an effect size that is in the same metric as, and thus comparable to, the d-statistic from between-groups designs. We expect it will be widely used.